The following examples are derived from *Homology manifold bordism* by Heather Johnston and Andrew Ranicki (Trans. Amer. Math. Soc. **352** no 11 (2000), PII: S 0002-9947(00)02630-1).

The results of Johnston [5] on homology manifolds are extended here. It is not possible to investigate transversality by geometric methods—as in [5] we employ bordism and surgery instead.

The proof of transversality is indirect, relying heavily on surgery theory—see Kirby and Siebenmann [7, III, §1], Marin [8] and Quinn [11]. We shall use the formulation in terms of topological block bundles of Rourke and Sanderson [12].

Q is a codimension q subspace by Theorem 4.9 of Rourke and Sanderson [12]. (Hughes, Taylor and Williams [4] obtained a topological regular neighborhood theorem for arbitrary submanifolds ....)

Wall [13, Chapter 11] obtained a codimension q splitting obstruction ....

... following the work of Cohen [2] on PL manifold transversality.

In this case each inverse image is automatically a PL submanifold of codimension σ (Cohen [2]), so there is no need to use s-cobordisms.

Quinn [10, 1.1] proved that ...

Theorem 3.1 (The additive structure of homology manifold bordism, Johnston [5]). ...

For $m \geq 5$ the Novikov-Wall surgery theory for topological manifolds gives an exact sequence (Wall [13, Chapter 10].

The surgery theory of topological manifolds was extended to homology manifolds in Quinn [9, 10] and Bryant, Ferry, Mio and Weinberger [1].

The 4-periodic obstruction is equivalent to an m-dimensional homology manifold, by [1].

Thus, the surgery exact sequence of [1] does not follow Wall [13] in relating homology manifold structures and normal invariants.

... the canonical TOP reduction ([3]) of the Spivak normal fibration of M ...

Theorem 3.2 (Johnston [5]). ...

Actually [5, (5.2)] is for $m \geq 7$, but we can improve to $m \geq 6$ by a slight variation of the proof as described below.

(This type of surgery on a Poincaré space is in the tradition of Lowell Jones [6].)

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